Quantum interference of latent time-correlations

Hlér Kristjánsson[1], Wenxu Mao[2,3] and Giulio Chiribella[2,1]

1. Department of Computer Science, University of Oxford, Oxford, United Kingdom
2. Department of Computer Science, The University of Hong Kong, Hong Kong
3. Cornell Tech, Cornell University, New York, NY, United States of America

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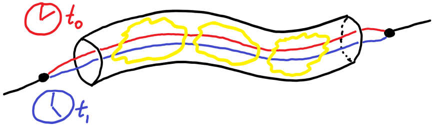
Introduction

- A series of recent papers have demonstrated communication advantages from placing communication channels in a quantum superposition of alternative configurations.
    1. Coherent control over the **causal ordering** of transmission lines[1,2,3]
    2. Coherent control over the **trajectory** through independent transmission lines[4,5,6].

- Here, demonstrate novel effects of coherent control over the **time of application** of a single time-correlated transmission line.



[1] D. Ebler *et al.*, *Phys. Rev. Lett.* **120**, 120502 (2018).
[2] S. Salek *et al.*, *arXiv:1809.06655* (2018).
[3] G. Chiribella *et al.*, *arXiv:1810.10457* (2018).
[4] N. Gisin *et al.*, *Phys. Rev. A* **72**, 012338 (2005).
[5] A. A. Abbott *et al.*, *arXiv:1810.09826* (2018).
[6] G. Chiribella, H. Kristjánsson, *Proc. R. Soc. A* **475**, 20180903 (2019).

Outline

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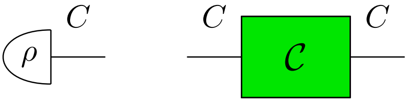
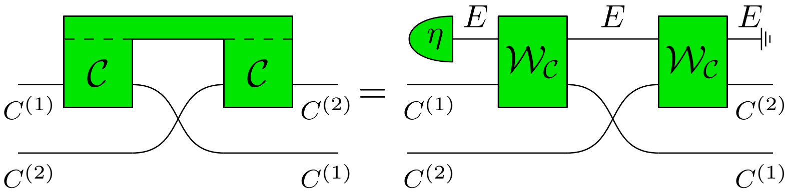
Notation: quantum states and channels

- Information carriers are described by **quantum states** $\rho \in \mathrm{St}(C)$ of a quantum system C, corresponding to Hilbert space $\mathcal{H}_C$.
- One use of a transmission line is described by a **quantum channel** $\mathcal{C} \in \mathrm{Chan}(C)$, i.e. a completely positive trace-preserving map from $\mathrm{St}(C)$ to $\mathrm{St}(C)$.
- Quantum channels act on quantum states as $\mathcal{C}(\rho) = \sum_{i=1}^{r} C_i \rho C_i^\dagger$, where $\{C_i\}_{i=1}^r$ is a non-unique set of **Kraus operators**.



Notation: time-correlated channels

- Multiple uses of a time-correlated transmission line are described by a **time-correlated quantum channel**[7] (quantum comb[8] or non-Markovian quantum channel[9]) $\mathcal{C}_{\text{cor}} \in \text{Chan}(C^{(1)} \otimes C^{(2)}, C^{(1)} \otimes C^{(2)})$, with no signalling from output $C^{(2)}$ to input $C^{(1)}$.
- Here consider the simple case where each use of $\mathcal{C}_{\text{cor}}$ in isolation is described by the same quantum channel $\mathcal{C}$.

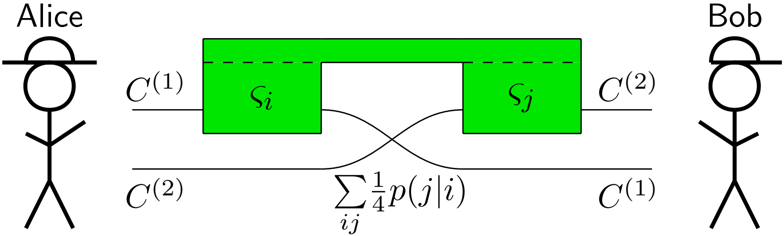


- Can be realised through two copies of a Stinespring dilation $\mathcal{W}_{\mathcal{C}}$ of $\mathcal{C}$ with an environment $|\eta\rangle_E$.

[7] C. Macchiavello, G. M. Palma, *Phys. Rev. A* **65**, 050301 (2002).
[8] G. Chiribella *et al.*, *EPL (Europhys. Lett.)* **83**, 30004 (2008).
[9] F. A. Pollock *et al.*, *Phys. Rev. A* **97**, 012127 (1 2018).

Communication through time-correlated channels

- Consider a time-correlated transmission line, where each use in isolation is described by a uniform randomisation over the identity and three Paulis $\{\varsigma_i(\cdot) = \sigma_i(\cdot)\sigma_i\}_{i=0}^{3}$, i.e. a completely depolarising channel $\mathcal{D} = \sum_{i=0}^{3} \frac{1}{4}\varsigma_i$.
- Communication at a non-zero rate is possible with an appropriate encoding over two particles[10].

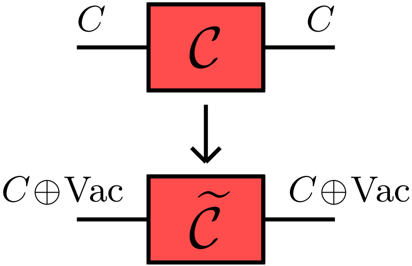


- Here, we achieve **perfect classical communication** by encoding only one bit on a **single particle in a superposition of times** $|\psi\rangle_M \otimes |+\rangle_T$.

[10] C. Macchiavello, G. M. Palma, *Phys. Rev. A* **65**, 050301 (2002).

Vacuum extension (1/2)

- Model communication devices as channels acting on the **vacuum state** $|\text{vac}\rangle$ in the sector Vac when no message is input[11].
- Overall, devices act on the extended system $\widetilde{C} := C \oplus \text{Vac}$.



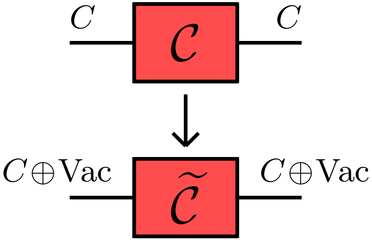
Definition

Channel $\widetilde{\mathcal{C}} \in \text{Chan}(\widetilde{C})$ is a **vacuum extension** of channel $\mathcal{C} \in \text{Chan}(C)$ if the action of $\widetilde{\mathcal{C}}$ on any state in sector C (Vac) is given by $\mathcal{C}$ ($\mathcal{I}_{\text{Vac}}$), where $\mathcal{I}_{\text{Vac}}$ is the identity on Vac.

[11] X.-Q. Zhou *et al.*, *Nat. Comm.* **2**, 413 EP (2011).

Vacuum extension (2/2)

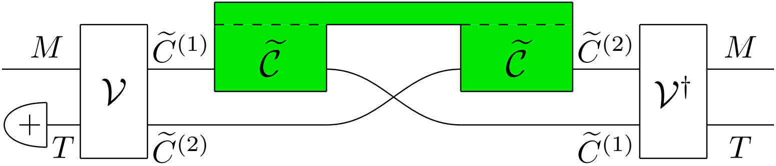
- The Kraus operators are $\widetilde{C}_i = C_i \oplus \gamma_i \ |\text{vac}\rangle\langle\text{vac}|$, where $\{\gamma_i\}_{i=1}^{r}$ are **vacuum amplitudes** satisfying $\sum_i |\gamma_i|^2 = 1$ [12].
- Mathematically, the vacuum extension is non-unique, however, the choice of vacuum extension is **uniquely determined** by the physics of the device.
- A time-correlated channel $\mathcal{C}_{\text{cor}}$ has a vacuum extension $\widetilde{\mathcal{C}}_{\text{cor}}$.



[12] G. Chiribella, H. Kristjánsson, *Proc. R. Soc. A* **475**, 20180903 (2019).

Superposition from vacuum extensions (1/2)

- The composite system $\widetilde{C}^{(1)} \otimes \widetilde{C}^{(2)}$ contains a **one-particle sector** $\left(C^{(1)} \otimes \mathrm{Vac}\right) \oplus \left(\mathrm{Vac} \otimes C^{(2)}\right)$.
- This is isomorphic to $M \otimes T$, where $M \simeq C^{(1)} \simeq C^{(2)}$ is the **message** system and T is the **timer** qubit of alternative times $|0\rangle_T$ and $|1\rangle_T$.



Superposition from vacuum extensions (2/2)

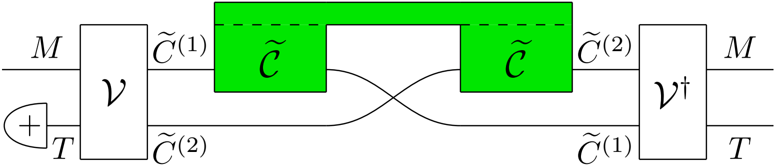
Definition

The **superposition of times** of $\mathcal{C}_{\text{cor}}$ **specified by the vacuum extension** $\widetilde{\mathcal{C}}_{\text{cor}}$ is the channel

$$\mathcal{S}(\widetilde{\mathcal{C}}_{\text{cor}})(\rho) := \mathcal{V}^{\dagger} \circ \widetilde{\mathcal{C}}_{\text{cor}} \circ \mathcal{V}\left(\rho \otimes |+\rangle\langle+|\right),$$

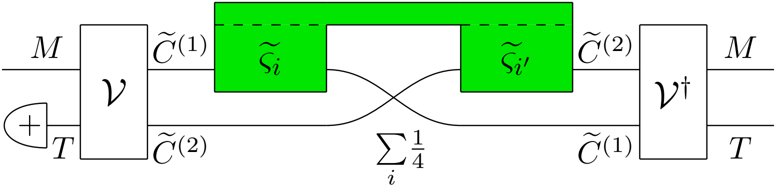
where the isomorphism $\mathcal{V} = V(\cdot)V^{\dagger}$ between the particle picture $M \otimes T$ and the mode picture $\left(C^{(1)} \otimes \text{Vac}\right) \oplus \left(\text{Vac} \otimes C^{(2)}\right)$ is defined by

$$V(|\psi\rangle_M \otimes |0\rangle_T) := |\psi\rangle_{\widetilde{C}^{(1)}} \otimes |\text{vac}\rangle_{\widetilde{C}^{(2)}}$$
$$V(|\psi\rangle_M \otimes |1\rangle_T) := |\text{vac}\rangle_{\widetilde{C}^{(1)}} \otimes |\psi\rangle_{\widetilde{C}^{(2)}}\,.$$



Perfect communication through white noise (1/3)

- Consider a time-correlated transmission line, where each use in isolation is described by a uniform randomisation over the identity and three Paulis $\{\varsigma_i(\cdot) = \sigma_i(\cdot)\sigma_i\}_{i=0}^{3}$, i.e. a completely depolarising channel $\mathcal{D} = \sum_{i=0}^{3} \frac{1}{4}\varsigma_i$.
- The second choice of unitary $\sigma_{i'}$ is correlated with the first choice of unitary σ_i via a permutation $\mathcal{P} : i \mapsto i'$, leading to the time-correlated channel $\mathcal{D}_{\mathcal{P}}$.
- The vacuum extension of each Pauli is given by $\widetilde{\sigma}_i = \sigma_i \oplus e^{i\phi_i} |\text{vac}\rangle\langle\text{vac}|$, with $\widetilde{\mathcal{D}} = \sum_{i=0}^{3} \frac{1}{4}\widetilde{\varsigma}_i$.

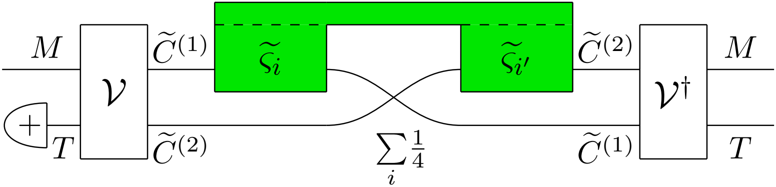


Perfect communication through white noise (2/3)

- The superposition of times of $\mathcal{D}_{\mathcal{P}}$ specified by the vacuum extension $\widetilde{\mathcal{D}}_{\mathcal{P}}$, where $\widetilde{\sigma}_i = \sigma_i \oplus e^{i\phi_i} |\text{vac}\rangle\langle\text{vac}|$, is

$$\mathcal{S}\left(\widetilde{\mathcal{D}}_{\mathcal{P}}\right)(\rho) = \frac{\frac{I}{d} + \mathcal{G}(\mathcal{P}, \{\phi_i\}, \rho)}{2} \otimes |+\rangle\langle+| + \frac{\frac{I}{d} - \mathcal{G}(\mathcal{P}, \{\phi_i\}, \rho)}{2} \otimes |-\rangle\langle-|,$$

  assuming the **latent interference term** $\mathcal{G}(\mathcal{P}, \{\phi_i\}, \rho) := \frac{1}{4}\sum_{i=0}^{3} e^{i(\phi_{i'} - \phi_i)} \sigma_i \rho \sigma_{i'}$ is hermitian.



- Depends both on the vacuum amplitudes $\{e^{\phi_i}\}$ and the permutation of noisy processes $\mathcal{P}$.
- The time-correlations are **latent** – accessible only via the interference of the time modes.

Perfect communication through white noise (3/3)

- For the permutation $\mathcal{P}(0,1,2,3) = (1,0,3,2)$, encode one bit of information in the orthogonal states $|\pm\rangle\langle\pm|$:

$$\mathcal{G}\left[(1,0,3,2),\{\phi_i\},|\pm\rangle\langle\pm|\right] = \frac{1}{2}\left\{|\pm\rangle\langle\pm|\cos\left(\phi_x - \phi_0\right) \pm |\mp\rangle\langle\mp|\sin\left(\phi_z - \phi_y\right)\right\}.$$

- For $\phi_z - \phi_y = \pi/2,\ \phi_x - \phi_0 = 0$, this gives $\mathcal{G}\left[(1,0,3,2),\{\phi_i\},|\pm\rangle\langle\pm|\right] = \pm I/2$. Hence, $\mathcal{S}\left(\widetilde{\mathcal{D}}_{(1,0,3,2)}\right)(|\pm\rangle\langle\pm|) = I/2_M \otimes |\pm\rangle\langle\pm|_T$, which is a noiseless bit channel with **unit classical capacity**.
- Cannot be achieved with the superposition of trajectories through independent quantum channels.

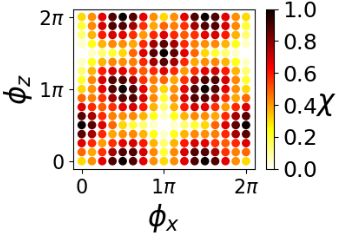
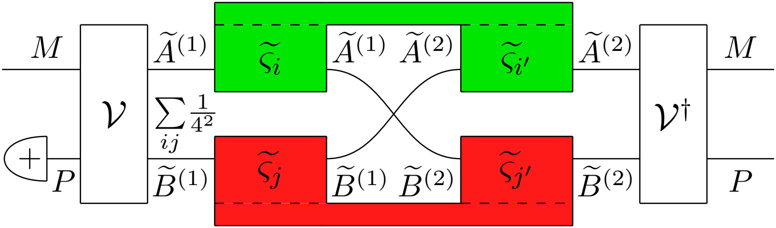


Figure 1: A plot of the parameters ϕ_x and ϕ_z of $\mathcal{S}\left(\widetilde{\mathcal{D}}_{(1,0,3,2)}\right)$ (on the two axes) against the Holevo information $\chi\left[\mathcal{S}\left(\widetilde{\mathcal{D}}_{(1,0,3,2)}\right)\right]$ (colour) for the superposition of times of $\mathcal{D}_{(1,0,3,2)}$ specified by the vacuum extensions $\widetilde{\sigma}_i = \sigma_i \oplus e^{i\phi_i}|\text{vac}\rangle\langle\text{vac}|$.

Simulation of the quantum SWITCH

- With two time-correlated transmission lines, can simulate the action of the quantum SWITCH[13] for the identity permutation $\mathcal{P}(0,1,2,3) = (0,1,2,3)$ (and any choice of vacuum extensions), achieving a Holevo information of **0.049**[14].
- Corresponds to the photonic experiments of the quantum SWITCH[15](?).

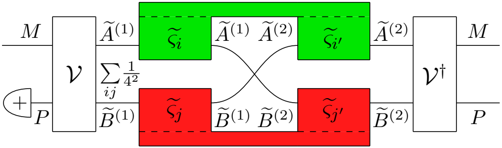


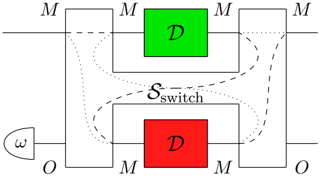
- In fact, we can do even better: the permutation $\mathcal{P}(0,1,2,3) = (1,0,3,2)$ can achieve a Holevo information of **0.311** (e.g. for $\phi_i = 0\ \forall i$).

[13] G. Chiribella *et al.*, *Phys. Rev. A* **88**, 022318 (2013).
[14] D. Ebler *et al.*, *Phys. Rev. Lett.* **120**, 120502 (2018).
[15] G. Rubino *et al.*, *Science Advances* **3**, e1602589 (2017).

Non-Markovianity vs indefinite causality (1/2)

- The resources[16][17] for the superposition of times vs the quantum `SWITCH` are different:
    - Superposition of times: **Two uses of two time-correlated (non-Markovian) transmission lines** with vacuum extensions, with coherent control of the times of application.
    - Quantum `SWITCH`: **One use of each of two independent transmission lines** (the specification of vacuum extensions not required), with coherent control over their causal order.
    - The former can simulate the latter, so defines a stronger set of resources.

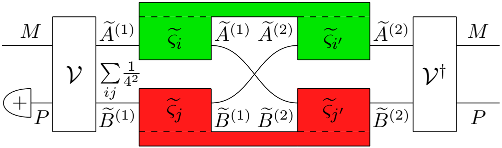


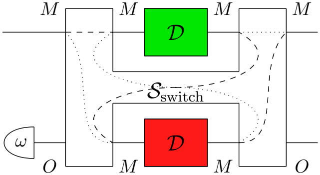


[16] P. A. Guérin *et al.*, *Phys. Rev. A* **99**, 062317 (2019).
[17] H. Kristjánsson *et al.*, *arXiv:1910.08197* (2019).

Non-Markovianity vs indefinite causality (2/2)

- The communication advantages in the superposition of times arise from the interplay between
  1. the **permutations** $\mathcal{P}$ of time-correlated noisy processes
  2. the **phase differences** $\{\phi_i\}$ between the action of each noisy process on the one-particle and vacuum sectors
  3. the **coherent control over the time of application**.
- The communication advantages of the quantum `SWITCH` arise solely from the **coherent control over the causal order**.





Summary and outlook

- Presented a novel quantum phenomenon: the **interference of latent time-correlations**.
- Constructed from a **superposition of time modes** through the **vacuum extension of time-correlated channels**.
- Enables perfect communication through time-correlated white noise, using only a **single particle in a superposition of time modes**.
- This framework can be used to **simulate the quantum** `SWITCH`.